Section 7.4

Decomposition of N(x)/D(x) Into Partial Fractions

- a) If $\deg(N(x)) \ge \deg(D(x))$, try long division and then use the method of partial fractions on the remainder.
- b) Completely factor the denominator into factors of the form $(px+q)^m$ and $(ax^2+bx+c)^n$ where $(ax^2+bx+c)^n$ is irreducible.
- c) For each factor of the form $(px + q)^m$, the partial fraction decomposition must include the following sum of m fractions.

$$\frac{A_1}{(px+q)} + \frac{A_2}{(px+q)^2} + \dots + \frac{A_m}{(px+q)^m}.$$

d) For each factor of the form $(ax^2 + bx + c)^n$, the partial fraction decomposition must include the following sum of n fractions.

$$\frac{B_1x+C_1}{(ax^2+bx+c)} + \frac{B_2x+C_2}{(ax^2+bx+c)^2} + \cdots + \frac{B_nx+C_n}{(ax^2+bx+c)^n}.$$

Problem 1. Write the partial fraction decomposition of the rational expression.

a)
$$\frac{1}{9x^2-16}$$

b)
$$\frac{-2x-2}{x^2-6x+8}$$

c)
$$\frac{x}{(x-1)(x^2+x+1)}$$

d)
$$\frac{2+x}{x^2-2x+1}$$

e)
$$\frac{x^2-x+1}{(x^2+1)^2}$$